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R E S E A R C H E S
INTO SOME PARTS OF THE
THEORY OF THE PLANETS,

In which is Solved the PROBLEM,
TO DETERMINE THE *CIRCULAR* ORBIT OF
A PLANET BY TWO OBSERVATIONS;

EXEMPLIFIED IN THE
NEW PLANET,

By *WALTER MINTO*.

L O N D O N :

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M,DCC,LXXXIII.

Rev. W. Monteith

From his humble servt



The Author

TO THE MOST ILLUSTRIOUS

JOSEPH SLOP *DE CADENBERG*,

DOCTOR OF LAWS, PROFESSOR OF ASTRONOMY IN

THE UNIVERSITY OF PISA, AND MEMBER OF

THE ACADEMY OF BONONIA.

Most Illustrious SIR,

THE other nations of Europe
ought to regard Italy with an
eye of particular gratitude. The
Sciences and the fine Arts and Com-
merce, which Superstition and the
inundations of Barbarians had almost
entirely destroyed, were revived

and greatly advanced by the noble efforts of your Countrymen.

'Tis to your GIOIA we owe the *Mariner's Compass*; to your COLUMBO, *America*; to your TORRICELLI, the *Barometer*, and to your great GALILEO, the *Law of the fall of Heavy Bodies*.

Italy can at this day boast of having given birth to a Mr de la G---GE, Director of a very celebrated Academy in Prussia, one of the most profound Mathematicians that ever appeared; and to a gentleman, greatly distinguished by his scientific knowledge and his amiable

qualities, whom delicacy hinders me from naming.

I am happy in having this opportunity of acknowledging in a public manner the obligations which I contracted to you during my long stay at Pisa. Among the many very considerable marks of friendship with which you honoured me, that of directing my Mathematical Studies was none of the least. For any little proficiency I may have made, I am indebted to You; and to You I think myself obliged in gratitude to dedicate the first fruits of my labours.

To merit the continuance of
your friendship and esteem shall be
the invariable endeavour of one,
who has the honour to be, with
the greatest respect,

Most Illustrious SIR,

Your most obliged,

And most devoted Servant,

EDINBURGH, }
Feb. 18. 1783. }

WALTER MINTO.

P R E F A C E.

WE owe many of the most important Discoveries in Nature and Art to Chance, and to Men of Professions totally unconnected with them.

Printing was invented by a Soldier, and Gunpowder by a Monk.

The Magnifying Powers of the Telescope have been greatly extended, and a NEW PLANET discovered, by Mr WILLIAM HERSCHEL, a Musician by profession.

We have no account of the Discovery of the other Planets. Being all visible to the naked eye, their appearance and motions, different from those of the fixed stars, would be remarked in the first ages of mankind.

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The Discovery of the NEW PLANET will form an Epoch in Astronomy. Although it is invisible to the naked eye, it may have been observed by different Astronomers since the invention of the Telescope; for it requires no great magnifying power to see it with sufficient distinctness: But it differs little in appearance from some fixed stars of an inferior magnitude, and has been probably mistaken for one of them.

While Mr HERSCHEL, engaged in a series of Observations, contrived in a most ingenious manner, on the Parallax of the fixed stars, was examining the small stars in the neighbourhood of H Geminorum, he perceived among them, in the evening of the 13th of March 1781, one which appeared visibly larger than the rest. Being struck with its uncommon magnitude, he compared it to H Geminorum, and the small star in the quartile between Auriga and Gemini,

and finding it so much larger than either of them, suspected it to be a *Comet*.

“ I knew,” says he, (PHIL. TRAN. 1781, Part II.) “ from experience, that the
 “ diameters of the fixed stars are not pro-
 “ portionally magnified with higher pow-
 “ ers as the planets are. I observed it
 “ with the magnifying powers 227, 460,
 “ and 932, and found the diameter of
 “ the comet encreased in proportion.”

All the astronomers of Europe were soon informed of the appearance of this New Star, and most of them have continued, as circumstances allowed, to observe it ever since.

The slowness of its motion ; its motion according to the order of the signs of the Zodiac ; its vicinity to the ecliptic, and some other circumstances, made them soon suspect it to be a *Planet*.

A memoir was read to the Royal Academy of Sciences and Belles Lettres at Berlin relative to this New Star, seven or eight months after its discovery ; and Mr De la LANDE read a memoir on the same subject, in the course of last winter, to the Royal Academy of Arts and Sciences at Paris, at a meeting in which the Grand Duke and Duchefs of Ruffia were present.

But the only work, of any consequence on this subject, that, as far as I know, has hitherto appeared, is the *Observationes et Theoria Novi Planetæ*, published at Pifa in the month of July last year by Mr SLOP. On account of the very great accuracy of his astronomical observations, and the scrupulous exactness of his calculations, joined to a profound knowledge of geometry, this Gentleman is deservedly ranked with the first of his profession.

The following sheets are divided into seven Sections.

The first Section contains part of the solution of the Problem, *To determine the orbit of a comet by three observations*, by Captain TEMPELHOFF of the King of Prussia's Artillery. This problem was proposed a few years ago by the Royal Academy of Arts and Belles Lettres at Berlin, and premiums were adjudged to the Marquis de CONDORCET and to Mr TEMPELHOFF for their solutions of it.

From the first Section are deduced in the second some methods for calculating the radii vectores of the Planets, the times of their periodical revolutions being supposed unknown. These methods were tried by Mr SLOP and myself, for determining the orbit of the New Planet; but our success did not at all answer our expectations: Our results in several different combinations differed widely from one another, and from the truth. They seem however not unworthy of a place, as some

of them may be of use to calculate the orbits of some Comets.

Mr SLOP, despairing of success from any known methods, found out an equation, by means of which he determined, with great accuracy, the radius vector of the New Planet at the time of its opposition to the Sun on 21st December 1781. A description of his equation, and the demonstration of a similar one are given in the third Section. In both these equations the radius vector is deduced from two observations, both very near, or one of them very near and the other at, the opposition. But as an error of even less than half a second of a degree, in observations so very near each other, must make a considerable error in the value of the radius vector, these equations can seldom be used to advantage. Mr SLOP's success depended on the great exactness of two of his own observations made in the neighbourhood of the opposition.

The two equations referred to led me to think, that by *any two* observations whatever the *circular* orbit of a Planet might be determined. This is demonstrated in Section IV. The Reader will easily see that these two equations are corollaries which may be drawn from the solution of the general Problem.

The fifth Section contains a number of observations of the New Planet, by Mr HERSCHEL at Bath, Mr SLOP at Pisa, Mr Professor ROBISON at Edinburgh, and others, from its first discovery on 13th March 1781 to 23d January 1783.

The observations of Mr HERSCHEL are calculated from the account which he has given of them in the Philosophical Transactions. In that account the Planet is traced in its motion through a group of Telescopic Stars, of which he gives figures and a scale. The different figures are connected by three interpolations, and the calculation proceeds on the supposition, that on March 13th

the Right Ascension of the Planet was $84^{\circ} 0' 0''$, and its North Declination $23^{\circ} 33' 0''$. I do not hear that any more exact determination has been given of these places.

The observations made at Mannheim by Mess. MAYER and KOENIG, Astronomers to the Elector Palatine, with an excellent mural quadrant of eight feet radius by *Bird*, seem, especially that of 1st February 1782, to be affected with considerable errors.

The Right Ascensions and Declinations observed by Mr SLOP were deduced from a comparison of the Planet with several telescopic stars in its neighbourhood. Those from the 5th October 1781 to 14th March 1782 inclusive, that of the 9th October excepted, were made with a mural quadrant of six feet radius by *Sisson*, and all the rest with an excellent reflecting telescope on an equatorial machine by *Short*. The longitudes and latitudes were com-

puted separately by Mr SLOP and myself, and, on a comparison, were found to agree to $\frac{1}{10}$ of a second, the longitude of 20th January excepted. The longitudes want the correction for the nutation.

The observations by Professor ROBISON are the intervals, in mean time, between the transits of the planet, and of a telescopic star, which he calls A, situated between η and μ Geminorum. It is found in the Britannic Catalogue, page 6th line 10th from the top. Its mean right ascension 1st January 1783, is nearly $91^{\circ} 24' 58''$, and its declination is $23^{\circ} 39' 17''$ north. The second column contains the difference of the declinations of the planet, and of H Geminorum. That very ingenious Gentleman does not consider these observations as of great moment, being made with no other instrument than a reflecting telescope furnished with a micrometer. The position of the hora-

ry wire was determined by means of a ruler making a certain angle with it, and so fixed that its edge covered two known fixed stars.

This section contains also the Sun's places, and the logarithms of its distances from the Earth for the mean times of most of the observations made at Pifa.

The sixth Section contains an illustration of the general problem of Section IV. by observations of the New Planet. From several different combinations the radius vector of its *circular* orbit, the time of its periodical revolution, and the inclination of its orbit to the ecliptic, are determined. It has been observed for too short a time, to enable us to determine its *elliptical* orbit with any tolerable degree of accuracy.

The last section contains Mr SLOP's theory, and its agreement with his obser-

vations. The longitudes are counted from the Planet's opposition to the Sun in the year 1781, December 21^d 18^h 38' 50" mean time at Pisa.

I hear Mr HERSCHEL has, with the approbation of the Royal Society, given to the New Planet, the name *Georgium Sidus*. The encouragement his present Majesty, by his beneficence and his own example, gives to astronomy, certainly entitles him more than any other living Sovereign to that honour. But it is not very probable that this name will be continued. The *Satellites* of *Jupiter* were named by their discoverer GALILEO, *Pianeti Medicei*, in honour of his patrons the MEDICI, a family, the memory of which will be revered as long as a taste for the fine arts and the sciences remains. This name, however, was discontinued.

Had Mr HERSCHEL or Mr SLOP given it the name of some of the ancient heathen

deities, it would have been, without hesitation, universally adopted. Among that number, *Minerva*, without all doubt, deserved the pre-eminence. The planet Venus has very properly obtained that name from its beauty and brilliancy; and the planet Mars has been so called from its red colour. The New Planet, being a telescopic star, may be said to denote the modesty of the Goddess of Wisdom.

Astronomers have agreed to distinguish each planet by a particular mark. Thus the mark for Jupiter is ♃, and that for Venus ♀. I have taken the liberty of denoting the New Planet by H, a compound of the first and last letters of the surname of its Discoverer.





RESEARCHES

INTO SOME PARTS OF THE

THEORY

OF THE

PLANETS, &c.

SECTION I.

Description of part of the solution of the Problem, To determine the orbit of a Comet by three observations.

THREE observations of a planet or comet being given, and calling

Its curt-distance from the earth, at the first, second and third observations x , y , and z .

Its geocentric latitudes λ , λ' and λ'' .

Its elongations from the sun e , e' and e'' .

The earth's distances from the sun a , b , and c ,

And the sun's motion from the first to the second observation, and from the first to the third d and d' .

Mr Tempelhoff rigidly demonstrates, that

$$\begin{aligned} \text{I. } & (\text{Tang. } \lambda \text{ fin. } \overline{e''-e'-d'+d} + \text{tang. } \lambda' \times \\ & \text{fin. } \overline{e-e''+d'} + \text{tang. } \lambda'' \text{ fin. } \overline{e'-e-d}) xyz + \\ & (\text{tang. } \lambda' \text{ fin. } \overline{e''-d'} - \text{tang. } \lambda'' \text{ fin. } \overline{e'-d}) ayz \\ & + (\text{tang. } \lambda'' \text{ fin. } \overline{e+d} - \text{tang. } \lambda \text{ fin. } \overline{e''-d'+d}) \\ & bxz + (\text{tang. } \lambda \text{ fin. } \overline{e'+d'-d} - \text{tang. } \lambda' \times \\ & \text{fin. } \overline{e+d'}) cxy - \text{tang. } \lambda'' \text{ fin. } d. abz + \text{tang. } \lambda' \times \\ & \text{fin. } d'. acy - \text{tang. } \lambda \text{ fin. } \overline{d'-d}. bcx = 0. \end{aligned}$$

The sectors described by the radius vector of a planet or comet are proportional to the times it takes to describe them. These sectors, if the motion of the heavenly body is slow, or the times short, will be

nearly proportional to triangles which have the same sides with the sectors, and the chords of their arcs for bases. Any two of these contiguous triangles are proportional to the parts of the chord of the arc of the two correspondent sectors determined by the intersection of that chord, and the side common to both triangles: Or, in other words, if (fig. I.) C C' C'' represent part of the orbit of a planet or comet, and SC, SC', SC'' its radii vectores, drawing the chord CEC'', and calling the time between the first and second observations and the time between the second and third m and n , we shall have $EC : EC'' :: m : n$,

This granted, our author proves, that

$$\begin{aligned} \text{II. } & \left(\frac{n}{m} a \sin. \overline{e'-d+c} \sin. \overline{e'+d'-d} \right) y - \frac{n}{m} x \\ & \sin. \overline{e'-e-d} . xy + \sin. \overline{e''-e'-d'+d} . yz - \frac{n}{m} b \times \\ & \sin. \overline{e+d} . x - b \sin. \overline{e''-d'+d} . z + b \left(\frac{n}{m} a \times \right. \\ & \left. \sin. \overline{d-c} \sin. \overline{d'-d} \right) = 0 \end{aligned}$$

By a very ingenious analysis, he shews likewise, that

$$\text{III. } (\text{Sin. } e' \text{ tang. } \lambda'' - \text{fin. } \overline{e'' - d' + d} \text{ tang. } \lambda'). \\ \cdot z - \frac{n}{m} (\text{fin. } \overline{e + d} \text{ tang. } \lambda' - \text{fin. } e' \text{ tang. } \lambda) \propto + \left(\frac{n}{m} a. \right. \\ \left. \cdot \text{fin. } d - c \text{ fin. } \overline{d' - d} \right) \text{ tang. } \lambda' = 0 \\ \text{or expressing the known quantities by } f, g, \text{ and } h,$$

$$\text{III. } fz - gx + h = 0.$$

If we call the radii vectores R, R' and R'' we have

$$\text{IV. } R^2 = a^2 - 2ax \text{ cofin. } e + x^2 \text{ sec.}^2 \lambda.$$

$$\text{V. } R'^2 = b^2 - 2by \text{ cofin. } e' + y^2 \text{ sec.}^2 \lambda'.$$

$$\text{VI. } R''^2 = c^2 - 2cz \text{ cofin. } e'' + z^2 \text{ sec.}^2 \lambda''$$

SECTION II.

*Some methods of determining the Radii vectores
of the Planets, deduced from Sect. I.*

IF the planet's orbit be supposed circular, the IV. V. and VI. equations, compared together, will give values for x and y , expressed by known quantities, and by z , which, substituted in equation I. or II. will free that equation from all unknown quantities, z excepted.

The value of z drawn from the III. equation, and substituted in equation VI. will, on a comparison of equations IV. and VI. produce a quadratic equation, where x only is the unknown quantity.

Since any three observations, at short intervals from each other, give

$$fz - gx + b = 0,$$

it is evident that, if the first and last of these observations are supposed to remain the same, and the middle one to be changed,

$$f'z - g'x + b' = 0;$$

four observations, therefore, will give the lineary equation,

$$x = \frac{fh' - f'h}{fg' - f'g}$$

If five observations are given: calling the correspondent curti-distances from the earth r, s, t, u, v , and taking, instead of f, g and b , the letters p, q and k , we have from

The first, second and third observations,

$$\text{I. } pt - qr + k = 0;$$

The first, second and fourth,

$$\text{II. } p'u - q'r + k' = 0;$$

The first, second and fifth,

$$\text{III. } p''v - q''r + k'' = 0.$$

The first of these equations, compared with the second, gives

$$u = \frac{pq't + q'k - qk'}{p'q}$$

and the first with the third,

$$v = \frac{pq''t + q''k - qk''}{p''q}$$

If we now take the third, fourth, and fifth observations, and make $t=x$, $u=y$ and $v=z$, and substitute the values of the two last in equation I. of Sect. I. we obtain a cubic equation, the solution of which will give the value of x . If the same values are substituted in equation II. of said section, we have a quadratic equation.

When the second of three observations is supposed made during the planet's opposition to the Sun, we have the angle $e'=180^\circ$, and, consequently,

$$\begin{aligned} & -\sin. e' - \overline{d' + d}. z - \frac{n}{m} \sin. e + \overline{d}. x + \frac{n}{m} a \sin. d - \\ & - c. \sin. \overline{d' - d} = 0, \end{aligned}$$

an equation free from every function of the planet's latitude ; a circumstance well worth noting, especially when that latitude is considerably small ; for a very little error in latitudes not surpassing a few minutes, must produce a very great one in an equation of this kind, where the tangents of these latitudes enter.

If therefore the second of five observations is supposed that of the planet's opposition to the Sun, and the values of the curti-distances at the third, fourth and fifth substituted in equation II. (of the former section) which also is free of all functions of the planet's latitude, one may very well be tempted to hope for a tolerably exact result from the solution of a quadratic equation.

It is hardly worth while to observe, that the curti-distances from the Earth being found, it is easy to calculate the radii

vectores and all the other elements of the orbit of a planet.

SECTION III.

Description of Mr Slop's equation for finding the Radius vector of a Planet supposed to describe a circle, and the demonstration of another equation of the same nature.

MR SLOP takes two interpolated places, the one twelve hours before, and the other twelve hours after the planet's opposition to the Sun, calls

The radius vector x

The Sun's distance from the Earth at the time of the opposition a

The Sun's mean diurnal motion at that time m

The Earth's diurnal motion at that same time n

The planet's observed diurnal geocentric motion q

The planet's heliocentric latitude λ

And demonstrates

$$x^3 - \frac{2a(n+q)x^2}{q \cdot \cos \lambda} + \frac{a^2(n+q)^2x}{q^2 \cos^2 \lambda} - \frac{m^2}{q^2} = 0$$

An equation of the same nature may be obtained in the following manner :

Suppose (fig. II.) the planet's place in the ecliptic at the time of its opposition to the sun in the point E' , and its place a few hours before or after, in the point E : let the Sun be in S , and the Earth in the points T and T' , corresponding to the places of the planet.

From the point S , through T' and E' , draw the indefinite straight line SG . Join S and E by the straight line SE , and S and T by ST . Draw TE and produce it till it meet SG in G . From the point T draw TF perpendicular to SG . Join the points E and E' ; it is evident that EE'

will be sensibly a straight line, and perpendicular to SG, consequently parallel to TF. This done, let

The Sun's distance from the Earth, $ST=a$,

The radius vector $=x$,

The planet's heliocentric latitude $=\lambda$,

Its elongation, $STG=T$,

The Sun's motion, $TSG=V$,

And the time between the observations supposed expressed in decimals of a year $=t$,

We have

$$SE=SE'=x \cos \lambda,$$

and the time of the planet's periodical revolution being $x\sqrt{x}$,

$$\text{The angle } ESE' = \frac{360^\circ t}{x\sqrt{x}}$$

and therefore

$$\text{The line } EE' = \frac{360^\circ t \cdot \cos \lambda}{\sqrt{x}}$$

$$TF = a \sin. V, \text{ and } SF = a \cosin. V,$$

$$SG = \frac{a \sin. V.}{\sin. T+V.} = x \cosin. \lambda + GE'$$

$$GE' = \frac{a \sin. V.}{\sin. T+V.} - x \cosin. \lambda.$$

$$FG = SG - SF = \frac{a \sin. V.}{\sin. T+V.} - a \cosin. V,$$

The similar triangles GEE and GFT give

$$GE' : EE :: GF : FT,$$

or substituting their values,

$$\frac{a \sin. V.}{\sin. T+V.} - x \cosin. \lambda : \frac{360^\circ. t. \cosin. \lambda}{\sqrt{x}}.$$

$$\therefore \frac{a \sin. V.}{\sin. T+V.} - a \cosin. V : a \sin. V.$$

therefore,

$$\frac{a \sin. T - \sin. T+V. \cosin. \lambda. x}{\sin. T - \cosin. V. \sin. T+V.} = \frac{360^\circ. t. \cosin. \lambda}{\sin. V. \sqrt{x}}.$$

or making

$$\frac{a \sin. T.}{\sin. T - \cosin. V. \sin. T+V.} = A,$$

$$\frac{\sin. \overline{T+V.}}{\sin. T - \cosin. V. \sin. \overline{T+V.}} = B.$$

and

$$\frac{360^\circ t.}{\sin. V.} = C.$$

We have the cubic equation

$$x^3 - \frac{2 A x^2}{B \cosin. \lambda} + \frac{A^2 x}{B^2 \cosin^2 \lambda} - \frac{C^2}{B^2} = 0.$$

similar to that of Mr Slop, and answering the same purpose.

In this, as well as in Mr Slop's equation, the heliocentric latitude is supposed known. If that latitude is small it may be taken at random. A first approximated value of x will give the necessary correction.

SECTION IV.

Solution of the Problem : To determine the circular orbit of a Planet from two observations,

LET the point S (fig. III.) represent the center of the Sun, the curve TT' part of the Earth's orbit, the curve PP' part of the orbit of a planet supposed to move in a circle, and the curve EE' part of the ecliptic. While the earth is supposed in T, let the planet be supposed observed in P; and while the Earth is in T' the Planet in P'. Draw the straight lines ST, ST', SP and SP'. From the points P and P' let fall perpendicular to the plane of ecliptic the straight lines PE and P'E'. Join the points S and E, S and E', T and E, T' and E'.

Let

The angles of elongation, $STE = T$, $ST'E' = T'$;

The angles formed by the intersections of the
curti-distances from the Sun $[SE, SE']$,
and the curti-distances from the Earth
 $[TE, T'E']$ in the points E and E', $SET = E$,
 $SE'T' = E'$;

And the Planet's motion in the ecliptic in the
interval of the observations $ESE' = \Sigma$.

Calling the Sun's longitude at the first ob-
servation $\odot$, and at the second $\odot'$;

And the correspondent geocentric longitudes
of the Planet P and P';

And making $P' - P = M$,

we have

$$M - E' + E - \Sigma = 0$$

For $T = \odot - P$, $T' = \odot' - P'$, the angle EST
 $= 180^\circ - \odot + P - E$, $E'ST' = 180^\circ - \odot' + P' -$
 $- E'$, and the angle $T'ST = \odot' - \odot$, substi-
tuted in the self-evident proposition

$$EST' = EST - T'ST = E'ST' - E'SE$$

give

$$180^\circ - \odot + P - E - \odot' + \odot = 180^\circ - \odot' + P' - E' - \Sigma,$$

which reduced produces the above equation.

In order to find the expressions of some functions of the arcs or angles E, E' and Σ ,

Let

The Earth's distance from the Sun at the first observation $= a$, and at the second $= a'$.

The correspondent heliocentric latitudes of the planet $= \lambda$ and λ' .

The radii vectores $SP = SP' = R$.

And the planet's motion in its orbit $PSP' = S$.

Join the points P and P', E and E' and parallel to EE' from the point P to where it meets P'E', draw the straight line PQ. PQ will be perpendicular to P'E'.

The right angled triangles PES and P'E'S give

$$SE = R. \cos \lambda; \text{ \& } SE' = R. \cos \lambda';$$

And the plain triangles SET and SE'T'

$$\text{Sin. } E = \frac{a \sin. T.}{R. \text{cofin. } \lambda}, \text{ and } \sin. E' = \frac{a' \sin. T'}{R. \text{cofin. } \lambda'}$$

$R\sqrt{R}$ being the time, expressed in years and decimals of years, of the periodical revolution of the planet; if we call t , expressed in the same manner, the time between the two observations: we have

$$S = \frac{360^\circ \cdot t}{R\sqrt{R}}.$$

The triangle ESE' gives

$$EE' : ES :: \sin. \Sigma : \sin. EE'S;$$

$$\text{And } EE' : E'S :: \sin. \Sigma : \sin. \overline{\Sigma + EE'S}.$$

therefore

$$\frac{\text{cofin. } \lambda}{\sin. EE'S} = \frac{\text{cofin. } \lambda'}{\sin. \Sigma + EE'S}$$

hence a value for $\sin. EE'S$, which substituted in the first proportion, with the necessary operations, produces

$$(EE')^2 = R^2 (\text{cofin. } \lambda - 2 \text{cofin. } \lambda. \text{cofin. } \lambda' \times \text{cofin. } \Sigma + \text{cofin. }^2 \lambda').$$

C

We have likewise

$$PQ = P'E' - PE = R (\sin. \lambda' - \sin. \lambda) = \\ = \sqrt{(P'P)^2 - (E'E)^2}$$

and

$$PP' = \frac{R \sin. S}{\sin. 90^\circ - \frac{1}{2}S} = 2 \cdot R \cdot \sin. \frac{1}{2}S$$

therefore

$$4R^2 \cdot \sin.^2 \frac{1}{2}S - R^2 (\cosin.^2 \lambda - 2 \cosin. \lambda \cdot \cosin. \lambda' \\ \times \cosin. \Sigma + \cosin.^2 \lambda') - R^2 (\sin. \lambda' - \sin. \lambda)^2 = 0.$$

hence

$$\cosin. \Sigma = \frac{1 - \sin. \lambda \cdot \sin. \lambda' - 2 \sin.^2 \frac{180^\circ + t}{R\sqrt{R}}}{\cosin. \lambda \cdot \cosin. \lambda'}$$

We have now got expressions for the sines of E and E', and for the cosine of Σ , in which the only unknown quantities are the radius vector and the sines and cosines of the heliocentric latitudes. The whole difficulty is therefore to find the expressions of these functions of the heliocentric latitudes.

For this purpose : Let

The geocentric latitudes of the Planet be
 l and l' ,

And its correspondent curti-distances from
the Earth, TE and $T'E' = v$ and v' .

In the right angled triangle PES we
have

$$PE = v \cdot \text{tang. } l = R \cdot \text{fin. } \lambda.$$

therefore

$$\text{fin. } \lambda = \frac{v}{R} \cdot \text{tang. } l.$$

$$\text{And cofin. } \lambda = \sqrt{1 - \frac{v^2}{R^2} \cdot \text{tang.}^2 l}.$$

The triangle SET gives

$$SE : ST :: \text{fin. } T : \text{fin. } E;$$

$$\text{And } SE : TE :: \text{fin. } T : \text{fin. } \overline{T+E},$$

where substituting for $\text{cofin. } \lambda$, $\text{fin. } E$, and
 $\text{cofin. } E$ their values, we obtain

$$R^2 = a^2 - 2av \cdot \text{cofin. } T + v^2 \cdot \text{fec.}^2 l,$$

which gives

$$v = \frac{a \cdot \text{cofin. } T}{\text{sec.}^2 l} + \sqrt{\frac{a^2 \text{cofin.}^2 T}{\text{sec.}^4 l} - \frac{a^2}{\text{sec.}^2 l} + \frac{R^2}{\text{sec.}^2 l}}$$

In the same manner it is demonstrated, that

$$v' = \frac{a' \cdot \text{cofin. } T'}{\text{sec.}^2 l'} + \sqrt{\frac{a'^2 \text{cofin.}^2 T'}{\text{sec.}^4 l'} - \frac{a'^2}{\text{sec.}^2 l'} + \frac{R^2}{\text{sec.}^2 l'}}$$

These expressions substituted respectively in the values of the fines and cofines of the heliocentric latitudes, and the new expressions resulting substituted in the formulas

$$\frac{a \cdot \text{fin. } T}{R \cdot \text{cofin. } \lambda} ; \quad \frac{a' \cdot \text{fin. } T'}{R \cdot \text{cofin. } \lambda'}$$

and

$$\frac{1 - \text{fin. } \lambda \cdot \text{fin. } \lambda' - 2 \text{fin.}^2 \frac{180^\circ \cdot t}{R\sqrt{R}}}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'}$$

will give expressions for the fines of E and E', and for the cofine of Σ , entirely free of every unknown quantity, the radius vector excepted.

Now as

$$\text{fin. } E = \frac{a \text{ fin. } T}{\sqrt{R^2 - v^2 \text{ tang. } l}} = E - \frac{E^3}{1.2.3} + \frac{E^5}{1.2.3.4.5} - \mathcal{O}^7 c$$

$$\text{fin. } E' = \frac{a' \text{ fin. } T}{\sqrt{R^2 - v'^2 \text{ tang. } l'}} = E' - \frac{E'^3}{1.2.3} + \frac{E'^5}{1.2.3.4.5} - \mathcal{O}^7 c.$$

$$\begin{aligned} \text{cofin. } \Sigma &= \frac{R^2 \left(1 - \frac{vv'}{R^2} \text{tang. } l \cdot \text{tang. } l' - 2 \text{ fin. } \frac{180^\circ l}{R\sqrt{R}} \right)}{\sqrt{(R^2 - v^2 \text{ tang. }^2 l)(R^2 - v'^2 \text{ tang. }^2 l')}} \\ &= 1 - \frac{\Sigma^2}{1.2} + \frac{\Sigma^4}{1.2.3.4} - \mathcal{O}^7 c. \end{aligned}$$

it is evident no algebraic expressions of E , E' , and Σ , can be had, except their fifth powers should be quantities infinitely little; and consequently the equation

$$M - E' + E - \Sigma = 0$$

cannot be solved in a direct manner.

COROLLARIES.

If the angle $T = 180^\circ$, which is the case if the first observation of the planet

is supposed made during its opposition to the Sun, we have

$$M - E' - \Sigma = 0,$$

or if $T = 180^\circ$,

$$M + E - \Sigma = 0,$$

and if both $T = 180^\circ$ and $T' = 180^\circ$,

$$M - \Sigma = 0,$$

which is of itself self-evident.

If the time between the observations is short, and the planet's heliocentric latitude encreases or diminishes slowly, we have

$$\Sigma = S,$$

and therefore

$$M - E' + E - S = 0,$$

and if both observations are near the

planet's opposition to the Sun, we have

$$\sin. E = E = \frac{a. \sin. T}{R. \cos. \lambda}$$

and

$$\sin. E' = E' = \frac{a'. \sin. T'}{R. \cos. \lambda}$$

and therefore

$$M - \frac{a' \sin. T' + a. \sin. T}{R. \cos. \lambda} - \frac{360^\circ. t}{R\sqrt{R}} = 0,$$

an equation of the third degree, the solution of which will give you a very exact value of R, provided (the observations being exact), you have first an approximated value to calculate the cosine of the planet's heliocentric latitude by.

If the geocentric longitudes are supposed equal, we have $M=0$; and consequently

$$-E' + E - \Sigma = 0.$$

this is the case with some before and some after the planet's being stationary. And if the time between the observations is short, the motion in latitude slow, and the planet at a distance great enough to make $\sin. E = E$, and $\sin. E' = E'$, the equation becomes lineary,

$$R - \left(\frac{360^\circ \cdot t \cdot \cos \lambda}{a \cdot \sin. T - a' \sin. T'} \right)^2 = 0.$$

To make use of the general equation

$$M - E' + E - \Sigma = 0.$$

Substitute an arbitrary number for R in the equations

$$v = \frac{a \cdot \sin. T}{\sec.^2 l} \pm \sqrt{\frac{a^2 \sin.^2 T}{\sec.^4 l} - \frac{a^2}{\sec.^2 l} + \frac{R^2}{\sec.^2 l}}$$

and

$$v' = \frac{a' \sin. T'}{\sec.^2 l'} \pm \sqrt{\frac{a'^2 \sin.^2 T'}{\sec.^4 l'} - \frac{a'^2}{\sec.^2 l'} + \frac{R^2}{\sec.^2 l'}}$$

and the values of v and v' thus found being substituted in the formulas

$$\frac{v}{R} \cdot \text{tang. } l, \quad \frac{v'}{R} \cdot \text{tang. } l'$$

will give you a value for the fines, and consequently for the cofines of λ and λ' : Then calculate the fines of E and E' , and you have the correspondent arcs. You may then calculate S , after which you will easily find the cofine, and consequently the arc Σ .

Substitute the arcs E , E' and Σ thus found in the general equation, which will become 0 if you have hit upon the true value of R , or will approach it in proportion as your supposed value approaches the real.

Suppose another value to R , and repeat the operation as before, and you will see which supposition leads you nearest the truth.

Let the first arbitrary value of R be r , and the second r' , and the equation, instead of 0, will, in the first case, become $=\omega$, for example, and in the second $=\omega'$. I say,

If the difference between the first and second values of R produces the difference $\omega - \omega'$ in the equation, what quantity to be subtracted from the first value of R will give the quantity ω .

Or, algebraically, calling the quantity sought m ,

$$r - r' : \omega - \omega' :: m : \omega$$

which gives

$$m = \frac{\omega(r - r')}{\omega - \omega'}$$

$$\text{And } R = r - m = \frac{\omega r' - \omega' r}{\omega - \omega'}$$

This value of R will be much nearer the true than any of the two former.

One or two operations more of the same kind will bring you to a value for R as near the true as can be desired.

If the latitude is only of a few minutes, some trouble may be saved by supposing

the heliocentric equal to the geocentric latitude, or even equal to 0, in order to obtain a first approximated value for R.

The formula

$$\frac{360^\circ \cdot t}{R\sqrt{R}}$$

will be expressed in seconds of a degree in reducing 360° to seconds. This gives

$$S = \frac{1296000'' \cdot t}{R\sqrt{R}}.$$

Let t'' be the number of seconds of time in t , we shall have t equal to t'' divided by the number of seconds in a year, or

$$t = \frac{t''}{31558151''}.$$

This substituted in the last expression of S gives

$$S = \frac{1296000'' \cdot t''}{31558151'' \cdot R\sqrt{R}} = \frac{0,040167 \cdot t''}{R\sqrt{R}}$$

and therefore

$$\text{Log. } S = 8,6134934 + \text{Log. } t'' - \text{Log. } R\sqrt{R}.$$

In order to find the place of the planet's Nodes, and the Inclination of its orbit to the ecliptic :

Let the portion of the planet's circular orbit, (fig. IV.) described in the interval of the two observations, be PP'. Produce P'P indefinitely, and describe part of a great circle of the ecliptic K'KN meeting P'P in the point N. From the points of the planet's places, P and P', let fall portions of great circles PK and P'K', perpendicular to K'KN.

We have

$$PK = \lambda, \quad P'K' = \lambda', \quad KK' = \Sigma,$$

$$P - E - KN = \text{Longitude of the planet's node,}$$

$$\text{And } PNK = \text{Inclination of its orbit.}$$

The spherical triangles PNK and P'NK' give

$$1 : \sin. KN :: \cotang. \lambda : \cotang. PNK,$$

$$1 : \sin. \Sigma + KN :: \cotang. \lambda' : \cotang. P'NK';$$

therefore

$$\cotang. KN = \frac{\cotang. \lambda - \cotang. \lambda' \cosin. \Sigma}{\cotang. \lambda' \sin. \Sigma}$$

SECTION V.

*Observations of the NEW PLANET : Sun's Places and
Logarithms of its distances from the Earth at the
times of some of these observations.*

RIGHT Ascensions and Declinations of the
New Planet, observed by

Mr HERSCHEL at Bath.

1781.

<i>Temp. ver. Baln.</i>				<i>Asc. Rect. H.</i>			<i>Dec. Bor. H.</i>		
<i>d.</i>	<i>h.</i>	<i>"</i>	<i>"</i>	<i>°</i>	<i>'</i>	<i>"</i>	<i>°</i>	<i>'</i>	<i>"</i>
Mar. 13.	10	30	0	84	0	0,0	23	33	0,0

Mr MASKELYNE at Greenwich.

<i>Temp. ver. Gren.</i>									
Mai. 28.				87	6	0,0	23	37	0,0

Mr SLOP at Pisa,

<i>Temp. ver. Pif.</i>									
Aug. 22.	16	12	4	92	11	12,8	23	40	13,3
23.	15	56	54	92	13	43,7	23	40	16,3
24.	16	5	57	92	16	8,7	23	40	21,7

1781.

		<i>Temp. ver. Pis.</i>				<i>Afc. Rect. H.</i>			<i>Afc. Bor. H.</i>		
		d.	h.			°	'	"	°	'	"
Aug.	26.	15	48	46		92	20	40,6	23	40	12,2
	27.	14	16	1		92	22	46,0	23	40	8,0
	28.	14	32	52		92	25	6,2	23	40	10,0
	29.	14	2	26		92	27	13,0	23	40	9,8
	30.	13	57	35		92	29	14,3	23	40	8,4
Sep.	1.	14	35	13		92	33	29,1	23	40	3,2
	2.	14	8	9		92	35	30,0	23	40	2,8
	5.	13	57	52		92	41	3,9	23	40	2,0
	11.	13	52	20		92	50	53,9	23	39	58,0
	15.	13	39	41		92	56	28,3	23	39	57,5
	18.	12	38	27		93	0	6,0	23	40	3,2
	24.	12	53	49		93	5	39,5	23	40	0,8
	29.	12	38	34		93	8	49,2	23	40	3,0
Oct.	5.	17	23	43		93	10	41,7	23	40	13,1
	6.	17	20	5		93	10	46,7	23	40	8,7
	8.	17	12	45		93	10	48,4	23	40	8,8
	9.	12	13	0		93	10	46,3	23	40	14,4
	14.	16	50	33		93	8	54,7	23	40	21,3
	18.	16	35	23		93	7	17,2	23	40	25,9
	24.	16	12	24		93	2	42,1	23	40	38,9

Mess. MAYER and KOENIG at Mannheim.

Temp. ver. Man.

Nov. 4. 15 28 40 | 92 49 19,9 | 23 41 31,0

Mr MASKELYNE.

Temp. ver. Gren.

8. | 92 42 20,0 | 23 41 30,0

Mr SLOP.

1781.

	<i>Temp. ver. Pis.</i>				<i>Asc. Rect. H.</i>			<i>Dec. Bor. H.</i>		
	d.	h.	'	"	°	'	"	°	'	"
Nov.	11.	10	36	3	92	37	20,0	23	41	22,2
Dec.	4.	13	19	4	91	44	39,1	23	42	36,2
	6.	13	9	58	91	39	21,8	23	42	41,9
	8.	13	0	51	91	33	59,4	23	42	51,9
	13.	12	37	58	91	20	14,6	23	43	3,1
	14.	12	33	22	91	17	25,6	23	43	6,4
	22.	11	56	23	90	54	57,3	23	43	15,1
	23.	11	51	50	90	52	8,8	23	43	15,3
	27.	11	33	22	90	40	57,6	23	43	20,9
	28.	11	28	47	90	38	15,6	23	43	23,5

1782.

Jan.	1.	11	10	22	90	27	6,7	23	43	29,2
	2.	11	5	48	90	24	28,4	23	43	30,6
	4.	10	56	42	90	18	56,1	23	43	30,7
	6.	10	47	36	90	13	34,2	23	43	33,2
	8.	10	38	32	90	8	14,8	23	43	33,9
	9.	10	34	1	90	5	32,6	23	43	33,1
	11.	10	25	1	90	0	27,6	23	43	34,8
	12.	10	20	34	89	57	57,3	23	43	32,2
	13.	10	16	7	89	55	22,4	23	43	34,2
	14.	10	11	38	89	52	54,9	23	43	33,8
	15.	10	7	11	89	50	31,4	23	43	33,3
	17.	9	58	21	89	45	53,0	23	43	30,8
	19.	9	49	34	89	41	3,2	23	43	33,0

1782.

	Temp. ver. Pis.				Asc. Rect. H.			Dec. Bor. H.		
	d.	h.	'	"	°	'	"	°	'	"
Jan.	20.	9	45	12	89	38	40,7	23	43	35,4
	22.	9	36	30	89	34	12,1	23	43	34,6
	28.	9	10	48	89	21	41,8	23	43	33,0
Feb.	1.	8	53	57	89	13	58,0	23	43	29,7
	4.	8	40	47	89	8	54,2	23	43	26,6
	11.	8	13	9	88	58	33,9	23	43	21,5
	15.	7	57	16	88	53	42,9	23	43	21,8
	16.	7	53	20	88	52	37,0	23	43	23,2
	18.	7	45	31	88	50	44,0	23	43	21,8
	19.	7	41	37	88	49	47,1	23	43	21,3
	20.	7	37	45	88	48	53,5	23	43	17,7
	26.	7	14	48	88	44	58,6	23	43	15,8
27.	7	11	2	88	44	41,5	23	43	11,2	
Mar.	3.	6	56	5	88	43	21,9	23	43	12,3
	4.	6	52	22	88	43	12,4	23	43	10,7
	5.	6	48	41	88	43	6,9	23	43	11,9
	6.	6	44	58	88	43	7,8	23	43	10,1
	8.	6	37	36	88	43	3,6	23	43	8,2
	10.	6	30	16	88	43	28,6	23	43	7,9
	11.	6	26	37	88	43	44,0	23	43	10,6
	13.	6	19	23	88	44	28,3	23	43	10,4
	14.	6	15	45	88	44	55,2	23	43	11,9

Mr MASKELYNE.

Temp. ver. Gren.

Oct. 24.

| 97 59 4,0 | 23 33 37,0

The geocentric Longitudes and Latitudes of the
NEW PLANET, computed from the Right Af-
censions and Declinations observed by

Mr HERSCHFL.

1781.

<i>Temp. ver. Baln.</i>				<i>Long. geoc. H.</i>			<i>Lat. Bor. H.</i>		
d.	h.	'	"	°	'	"	°	'	"
Mar. 13.	10	30	0	84	30	5,0	0	11	42,0
17.	11	0	0	84	32	53,0			
21.	10	0	0	84	35	58,0			
24.	8	12	0	84	39	15,0			
26.	10	43	0	84	41	28,0			
28.	7	46	0	84	44	5,0			
29.	8	50	0	84	45	33,0			
Apr. 1.	7	45	0	84	49	55,0			
15.	10	18	0	85	13	45,0			
16.	10	47	0	85	15	27,0			
19.	8	18	0	85	22	11,0	0	13	33,0

Mr DE CESARIS at Milan.

Temp. med. Pis.

Mai. 15. 8 5 52 | 86 37 46,8 | 0 11 36,7

Mr MASKELYNE.

Temp. ver. Gren.

28. | 87 20 35,0 | 0 10 25,0

E

1782.

	Temp. ver. Pis.				Asc. Rect. H.			Dec. Bor. H.		
	d.	h.	"		°	'	"	°	'	"
Jan.	20.	9	45	12	89	38	40,7	23	43	35,4
	22.	9	36	30	89	34	12,1	23	43	34,6
	28.	9	10	48	89	21	41,8	23	43	33,0
Feb.	1.	8	53	57	89	13	58,0	23	43	29,7
	4.	8	40	47	89	8	54,2	23	43	26,6
	11.	8	13	9	88	58	33,9	23	43	21,5
	15.	7	57	16	88	53	42,9	23	43	21,8
	16.	7	53	20	88	52	37,0	23	43	23,2
	18.	7	45	31	88	50	44,0	23	43	21,8
	19.	7	41	37	88	49	47,1	23	43	21,3
	20.	7	37	45	88	48	53,5	23	43	17,7
	26.	7	14	48	88	44	58,6	23	43	15,8
	27.	7	11	2	88	44	41,5	23	43	11,2
Mar.	3.	6	56	5	88	43	21,9	23	43	12,3
	4.	6	52	22	88	43	12,4	23	43	10,7
	5.	6	48	41	88	43	6,9	23	43	11,9
	6.	6	44	58	88	43	7,8	23	43	10,1
	8.	6	37	36	88	43	3,6	23	43	8,2
	10.	6	30	16	88	43	28,6	23	43	7,9
	11.	6	26	37	88	43	44,0	23	43	10,6
	13.	6	19	23	88	44	28,3	23	43	10,4
	14.	6	15	45	88	44	55,2	23	43	11,9

Mr MASKELYNE.

Temp. ver. Gren.
 Oct. 24. | 97 59 4,0 | 23 33 37,0

The geocentric Longitudes and Latitudes of the
NEW PLANET, computed from the Right Af-
censions and Declinations observed by

Mr HERSCHFL.

1781.

<i>Temp. ver. Baln.</i>				<i>Long. geoc. H.</i>			<i>Lat. Bor. H.</i>		
d.	h.	"	"	°	'	"	°	'	"
Mar. 13.	10	30	0	84	30	5,0	0	11	42,0
	17.	11	0	84	32	53,0			
	21.	10	0	84	35	58,0			
	24.	8	12	84	39	15,0			
	26.	10	43	84	41	28,0			
	28.	7	46	84	44	5,0			
	29.	8	50	84	45	33,0			
Apr. 1.	7	45	0	84	49	55,0			
	15.	10	18	85	13	45,0			
	16.	10	47	85	15	27,0			
	19.	8	18	85	22	11,0	0	13	33,0

Mr DE CESARIS at Milan.

Temp. med. Pis.

Mai. 15. 8 5 52 | 86 37 46,8 | 0 11 36,7

Mr MASKELYNE.

Temp. ver. Gren.

28. | 87 20 35,0 | 0 10 25,0

Mr MESSIER at Clugny.

1781.

<i>Temp. med. Pis.</i>				<i>Long. geoc. H</i>			<i>Lat. Bor. H</i>		
d.	h.	"	"	°	'	"	°	'	"
July 17.	15	49	15	89	42	22,0	0	12	1,3

Mr SLOP.

Aug. 22.	16	14	21	92	0	10,2	0	12	55,5
23.	15	58	55	92	2	28,4	0	13	0,6
24.	16	7	42	92	4	41,1	0	13	8,1
26.	15	49	58	92	8	50,2	0	13	2,6
27.	14	16	57	92	10	45,1	0	13	0,2
28.	14	33	30	92	12	53,5	0	13	4,4
29.	14	1	47	92	14	49,6	0	13	6,2
30.	13	56	38	92	16	40,7	0	13	6,7
Sept. 1.	14	34	38	92	20	34,1	0	13	5,5
2.	14	7	16	92	22	25,1	0	13	7,1
5.	13	55	59	92	27	30,6	0	13	12,0
11.	13	48	24	92	36	31,0	0	13	18,4
15.	13	34	22	92	41	37,2	0	13	24,0
18.	12	32	7	92	44	56,4	0	13	33,8
24.	12	45	25	92	50	1,9	0	13	37,9
29.	12	28	31	92	52	55,5	0	13	43,8
Oct. 5.	17	11	46	92	54	38,3	0	13	56,1
6.	17	7	51	92	54	43,1	0	13	51,8
8.	16	59	58	92	54	44,6	0	13	51,9
9.	12	0	1	92	54	42,5	0	13	57,5
18.	16	20	25	92	51	30,8	0	14	4,8
24.	15	56	36	92	47	18,6	0	14	12,4

Mess. MAYER and KOENIG.

1781.

<i>Temp. ver. Man.</i>				<i>Long. geoc. H</i>			<i>Lat. Bor. H</i>		
d.	h.	'	"	°	'	"	°	'	"
Nov. 4.	15	28	40	92	35	4,0	0	14	48,0

Mr MASKELYNE.

Temp. ver. Gren.

8.	92	28	39,0	0	14	43,0
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Mr SLOP.

Temp. med. Pis.

11.	10	20	26	92	24	4,1	0	14	28,1
Dec. 4.	13	10	6	91	35	49,0	0	14	58,3
6.	13	1	51	91	30	58,4	0	15	0,5
8.	12	53	37	91	26	3,2	0	15	7,1
13.	12	32	5	91	13	27,9	0	15	10,8
14.	12	28	58	91	10	53,1	0	15	12,7

Mess. MAYER and KOENIG.

Temp. ver. Man.

21.	12	1	12	90	53	8,0	0	15	30,0
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Mr SLOP.

1781.

	<i>Temp. med. Pis.</i>				<i>Long. geoc. H.</i>			<i>Lat. Bor. H.</i>		
	d.	h.	'	"	°	'	"	°	'	"
Dec. 22.	11	55	58		90	50	18,7	0	15	11,9
23.	11	51	55		90	47	44,5	0	15	11,1
27.	11	35	25		90	37	30,0	0	15	13,4
28.	11	31	19		90	35	1,6	0	15	15,3

1782.

Jan. 1.	11	14	48		90	24	49,2	0	15	18,7
2.	11	10	3		90	22	24,3	0	15	19,6
4.	11	2	31		90	17	20,1	0	15	18,9
6.	10	54	18		90	12	25,4	0	15	20,9
8.	10	46	5		90	7	32,9	0	15	21,2
9.	10	41	58		90	5	4,5	0	15	20,3
11.	10	33	46		90	0	25,2	0	15	21,9
12.	10	29	41		89	58	7,6	0	15	19,1
13.	10	25	37		89	55	45,8	0	15	21,4
14.	10	21	29		89	53	30,8	0	15	21,0
15.	10	17	23		89	51	19,5	0	15	20,7
17.	10	9	13		89	47	4,6	0	15	18,6
19.	10	1	3		89	42	39,3	0	15	21,3
20.	9	56	58		89	40	28,8	0	15	24,0
22.	9	48	48		89	36	22,9	0	15	23,9
28.	9	24	23		89	24	56,0	0	15	24,7

Mess. MAYER and KOENIG.

Temp. ver. Man.

Feb. 1. 8 54 7 | 88 55 2,0 | 0 15 38

Mr SLOP.

1782.

	<i>Temp. med. Pis.</i>				<i>Long. geoc. H.</i>			<i>Lat. Bor. H.</i>		
	d.	h.	"	"	°	'	"	°	'	"
Feb.	1.	9	8	6	89	17	51,5	0	15	23,5
	4.	8	55	14	89	13	13,3	0	15	22,0
	11.	8	27	49	89	3	45,4	0	15	20,6
	15.	8	11	48	88	59	18,9	0	15	22,8
	16.	8	7	47	88	58	18,7	0	15	24,6
	18.	7	59	48	88	56	33,3	0	15	24,0
	19.	7	55	48	88	55	43,1	0	15	24,0
	20.	7	51	49	88	54	53,9	0	15	20,7
	26.	7	27	57	88	51	18,9	0	15	20,7
	27.	7	24	0	88	51	3,2	0	15	16,2
Mar.	3.	7	8	13	88	49	50,4	0	15	18,0
	4.	7	4	16	88	49	41,6	0	15	16,5
	5.	7	0	21	88	49	36,7	0	15	17,7
	6.	6	56	24	88	49	37,4	0	15	15,8
	8.	6	48	32	88	49	33,6	0	15	14,0
	10.	6	40	40	88	49	56,5	0	15	13,5
	11.	6	36	45	88	50	10,6	0	15	16,1
	13.	6	28	58	88	50	51,1	0	15	15,5
	14.	6	25	3	88	51	15,8	0	15	16,8
<i>Temp. ver. Pis.</i>										
Aug.	20.	14	13	25	96	11	49,9	0	16	6,8
	21.	14	27	40	96	14	22,2	0	15	57,7
Sept.	3.	14	41	46	96	43	37,1	0	16	13,9
	4.	14	20	23	96	45	32,8	0	16	16,4

1782.

<i>Temp. ver. Pis.</i>				<i>Long. geoc. H</i>			<i>Lat. Bor. H</i>		
	d.	h.	"		°	'		°	'
Sept. 12.	13	34	3	96	59	26,3	0	16	20,8
13.	13	21	27	97	1	0,5	0	16	36,1
25.	13	5	14	97	15	20,1	0	16	54,9
26.	13	32	47	97	16	32,4	0	17	2,0

Mr MASKELYNE.

Temp. ver. Gren.

Oct. 24. | 97 18 55,0 | 0 17 36,0

Differences in mean time of the Transit of A
Geminorum and the NEW PLANET, and
differences of its Declination and that of
H Geminorum, observed at Edinburgh by

Mr Professor ROBISON,

1782.

<i>Tem. med. Edin.</i>				<i>Tr. H. post A.</i>			<i>D. H. — D.</i>	
	d.	h.	"		h.	"	°	'
Dec. 4.	8	51	20	0	21	25	0	22
9.	9	12	18	0	20	39		
14.	8	11	43	0	19	56		
15.	8	20	4	0	19	35		
18.	10	2	14	0	19	4		

		782.											
		<i>Tem. med. Edin.</i>				<i>Tr. H. post A.</i>				<i>D. H.—D. H</i>			
		d.	h.	"	"	h	"	o	"	o	"		
16 20,8 16 36,1 16 54,9 17 2,0	Dec.	21.	7	58	51	o	18	30		o	24	18	
		24.	9	24	o	o	17	57					
		26.	8	41	10	o	17	35					
		28.	8	34	35	o	17	13					
		30.	7	48	17	o	16	51					
		31.	9	52	8	o	16	40					
		783.											
17 36,6	Jan.	1.	6	58	42	o	16	31					
		2.	8	18	29	o	16	17					
		5.	10	15	15	o	15	44		o	25	40	
		8.	8	53	42	o	15	09					
		10.	8	51	33	o	14	49					
		11.	9	27	46	o	14	37 $\frac{1}{2}$					
		18.	8	36	o	o	13	26					
fit of A ET, and that of burgh by		19.	10	26	30	o	13	16					
		23.	8	12	39	o	12	39					

D. H.—D.
o
o 22 3

Longitudes of the Sun and Logarithms of its distances from the Earth for the mean times of the observations made at Pisa from the 22d August 1781 to 14th March 1782, inclusive.

1781.	d.	s	o	Long. $\odot$	Log. dist. $\odot$ at
Aug.	22.	5	0	16 5,3	0,0045029
	23.	5	1	13 25,0	0,0044036
	24.	5	2	12 44,7	0,0043016
	26.	5	4	7 2,2	0,0040953
	27.	5	5	1 17,4	0,0039945
	28.	5	6	0 2,6	0,0038874
	29.	5	6	56 52,9	0,0037806
	30.	5	7	54 46,8	0,0036735
Sept.	1.	5	9	52 33,0	0,0034539
	2.	5	10	49 37,4	0,0033447
	5.	5	13	43 53,2	0,0030160
	11.	5	19	33 57,3	0,0023435
	15.	5	23	27 42,1	0,0018793
	18.	5	26	21 16,2	0,0015207
	24.	6	2	14 51,9	0,0007653
	29.	6	7	8 9,4	0,0001290
Oct.	5.	6	13	15 50,4	9,9993484
	6.	6	14	14 58,9	9,9992284
	8.	6	16	13 23,1	9,9989857
	9.	6	17	0 25,7	9,9988880
	18.	6	26	7 43,8	9,9977754
	24.	7	2	5 53,3	9,9970391

1781.	d.	s	o	Long. °	Log. dist. ° a
Nov.	11.	7	19	55 28,1	9,9951538
Dec.	4.	8	13	19 49,2	9,9933501
	6.	8	15	21 19,1	9,9932491
	8.	8	17	22 59,2	9,9931604
	13.	8	22	27 27,6	9,9929605
	14.	8	23	28 16,4	9,9929267
	22.	9	1	36 13,5	9,9927045
	23.	9	2	37 12,5	9,9926869
	27.	9	6	41 5,9	9,9926334
	28.	9	7	42 4,2	9,9926287
1782.					
Jan.	1.	9	11	45 57,0	9,9926351
	2.	9	12	46 56,2	9,9926430
	4.	9	14	48 52,7	9,9926708
	6.	9	16	50 49,7	9,9927074
	8.	9	18	52 54,1	9,9927530
	9.	9	19	53 46,4	9,9927789
	11.	9	21	55 33,5	9,9928336
	12.	9	22	56 41,7	9,9928640
	13.	9	23	57 39,5	9,9928974
	14.	9	24	58 36,7	9,9929310
	15.	9	25	59 33,2	9,9929656
	17.	9	28	1 24,3	9,9930392
	19.	10	0	3 11,4	9,9931285
	20.	10	1	4 3,2	9,9931632
	22.	10	3	5 43,6	9,9932556
	28.	10	9	10 16,2	9,9935873
Feb.	1.	10	13	12 56,0	9,9938633

1782.	d.	s	o	Long. $\odot$	Long. dist. $\odot$ at
Feb.	4.	10	16	14 44,0	9,9940954
	11.	10	23	18 28,4	9,9946902
	15.	10	27	20 6,6	9,9950537
	16.	10	28	20 26,8	9,9951466
	18.	11	0	21 1,3	9,9953337
	19.	11	1	21 15,6	9,9954298
	20.	11	2	21 28,4	9,9955273
	26.	11	8	22 0,5	9,9961429
	27.	11	9	21 59,2	9,9962515
Mar.	3.	11	13	21 36,6	9,9967037
	4.	11	14	21 26,6	9,9968212
	5.	11	15	21 15,6	9,9969400
	6.	11	16	21 2,4	9,9970599
	8.	11	18	20 32,2	9,9973011
	10.	11	20	19 55,6	9,9975436
	11.	11	21	19 34,7	9,9976643
	13.	11	23	18 47,6	9,9979059
	14.	11	24	18 21,3	9,9980268

SECTION VI.

The general Problem of Section IV. Exemplified in the NEW PLANET.

TO illustrate our Problem, let us take the first and the last of Mr Slop's observations made at the Mural Quadrant.

The mean times at Pifa were

	d.	h.	'	"
1781. October 5.	17	11	46.	
1782. March 14.	6	25	3.	

The interval between the observations is therefore

d.	h.	'	"
159	13	13	17,

which reduced to seconds gives

$$t'' = 13785197''$$

$$\text{Log. } t'' = 7,1394131.$$

The Sun's places at the above periods were

$$\odot = \overset{s}{6} \overset{o}{13} \overset{'}{15} \overset{''}{50,4};$$

$$\odot' = 11 \ 24 \ 18 \ 21,3.$$

And supposing the Earth's mean distance from the Sun = 1, the logarithms of the Earth's distances from the Sun at the same periods are,

$$\text{Log. } a = 9,9993484;$$

$$\text{Log. } a' = 9,9980268.$$

The geocentric Longitudes of the planet corrected by the nutation were

$$P = \overset{s}{3} \overset{o}{2} \overset{'}{54} \overset{''}{30,4};$$

$$P' = 2 \ 28 \ 51 \ 10,4;$$

hence the Planet's Elongations

$$T = \odot - P = \overset{s}{3} \overset{o}{10} \overset{'}{21} \overset{''}{20,0};$$

$$T' = \odot' - P' = 8 \ 25 \ 27 \ 10,9.$$

The logarithms of the fines of these elongations are

$$\text{Log. fin. } T = 9,9928676;$$

$$\text{Log. fin. } T' = 9,9986310.$$

The planet's motion in geocentric longitude was

$$M = P' - P = 4^{\circ} 3' 20,0''.$$

The planet's geocentric Latitudes were

$$l = 0^{\circ} 13' 56,1'';$$

$$l' = 0^{\circ} 15' 16,8''.$$

The logarithms of the tangents and of the squares of the secants of these latitudes are

$$\text{Log. tang. } l = 7,6077645;$$

$$\text{Log. tang. } l' = 7,6476682;$$

$$\text{Log. fec.}^2 l = 10,0000071;$$

$$\text{Log. fec.}^2 l' = 10,0000083.$$

The geocentric latitudes being small, the cosines of the correspondent heliocentric latitudes will differ little from one another and from unity. To save trouble, therefore, we shall, in order to have a first approximated value for R , suppose $\cos \lambda = \cos \lambda' = 1$, and $\Sigma = S$.

This gives us

$$\text{Log. fin. } E = \log. \frac{a \text{ fin. } T}{R} = 9,9922160 - \log. R,$$

$$\text{Log. fin. } E' = \log. \frac{a' \text{ fin. } T'}{R} = 9,9966578 - \log. R,$$

$$\text{Log. } S = \log. \frac{0,040167 \cdot t''}{R\sqrt{R}} = 5,7529065 - \log. R\sqrt{R}.$$

The quantities M and t'' compared together, shew that our planet must be far beyond the most distant of any with which we are acquainted in our system. Let us begin then with supposing the radius vector $R = 15$ times the Sun's mean distance from the Earth, which is about one third

greater than the distance of Saturn. We have

$$\text{Log. R} = 1,1760913;$$

$$\text{Log. R}\sqrt{\text{R}} = 1,7641369;$$

consequently

$$\text{Log. fin. E} = 8,8161247.$$

$$\text{Log. fin. E}' = 8,8205665.$$

$$\text{Log. S} = 3,9887696.$$

therefore

$$\begin{array}{r} \overset{0}{=} \\ +M = -4 \quad 3 \\ -E' = +3 \quad 47 \\ +E = +3 \quad 45 \\ -S = -2 \quad 42 \\ \hline \omega = +0 \quad 47 \end{array}$$

here we are far from the truth, for the sum ω of the terms of our equation, instead of being $=0$ is $=47'$. It is evident, therefore, that R has been taken by far too little. Let us now suppose

$$\text{R} = 20,$$

we have

$$\text{Log. R} = 1,3010300.$$

$$\text{Log. R}\sqrt{\text{R}} = 1,9515450.$$

$$\text{Log. fin. E} = 8,6911860.$$

$$\text{Log. fin. E}' = 8,6956278.$$

$$\text{Log. S} = 3,8013615.$$

therefore

$$\begin{array}{r} \text{+M} = -4 \quad 3 \quad 20 \\ \text{-E}' = +2 \quad 50 \quad 38 \\ \text{+E} = +2 \quad 48 \quad 54 \\ \text{-S} = -1 \quad 45 \quad 29 \\ \hline \omega = -0 \quad 9 \quad 17 \end{array}$$

here it is plain R has been taken too great, but that it is nearer the truth than the former value, because $9' 17''$ are nearer to 0 than $47'$. Let

$$\text{R} = 19,05$$

we have

$$\text{Log. R} = 1,2798950.$$

$$\text{Log. } R\sqrt{R} = 1,9198425.$$

$$\text{Log. sin. } E = 8,7123210.$$

$$\text{Log. sin. } E' = 8,7167628.$$

$$\text{Log. } S = 3,8330640.$$

therefore

$$\begin{array}{r} \text{+M} = -4 \quad 3 \quad 20 \\ \text{+E'} = +2 \quad 59 \quad 9 \\ \text{+E} = +2 \quad 57 \quad 20 \\ \text{+S} = +1 \quad 53 \quad 29 \\ \hline \omega' = -0 \quad 0 \quad 20 \end{array}$$

We are now very near an approximated value for R . In order to come at it the sooner, I say: If the difference between the two last values of R , ($0,95$), give the difference between the two last values of ω , ($8' 57''$); what quantity, m , to be subtracted from the penult value of R , will give the penult value of ω , ($9' 17''$)? I find $m = 0,985$, &c. and therefore

$$R = 19,0155178.$$

G

$$\text{Log. } R = 1.2791095.$$

$$\text{Log. } R\sqrt{R} = 1,9186643.$$

$$\text{Log. fin. } E = 8,7131065.$$

$$\text{Log. fin. } E' = 8,7175483.$$

$$\text{Log. } S = 3,8342422.$$

therefore

$$\begin{array}{r} \text{+M} = -4^{\circ} 3' 20'' \\ \text{—E'} = +2^{\circ} 59' 28'' \\ \text{+E} = +2^{\circ} 57' 39'' \\ \text{—S} = -1^{\circ} 53' 47'' \\ \hline \omega = 0^{\circ} 0' 0'' \end{array}$$

With this value of R I find the Planet's aberration at the first observation = $-0'',5$, and at the second = $-2'',6$, and therefore

$$\begin{array}{r} \text{P} = +3^{\circ} 2' 54'' 29,9 \\ \text{P'} = +2^{\circ} 28' 51'' 7,8 \\ \text{M} = -0^{\circ} 4' 3'' 22,1 \\ \text{T} = +3^{\circ} 10' 21'' 20,5 \\ \text{T'} = +8^{\circ} 24' 27'' 13,5 \end{array}$$

[51]

$$\text{Log. fin. } T = 9,9928674.$$

$$\text{Log. fin. } T' = 9,9986314.$$

$$\text{Log. cofin. } T = 9,2546897.$$

$$\text{Log. cofin. } T' = 8,8990730.$$

$$\text{Log. } \frac{a \text{ cofin. } T}{\text{sec.}^2 l} = 9,2540410;$$

$$\text{Log. } \frac{a^2 \text{ cofin.}^2 T}{\text{sec.}^4 l} = 8,5080820;$$

$$\text{Log. } \frac{a^2}{\text{sec.}^2 l} = 9,9986997;$$

$$\text{Log. } \frac{R^2}{\text{sec.}^2 l} = 2,5582219;$$

$$\frac{a \text{ cofin. } T}{\text{sec.}^2 l} = -0,1794903;$$

$$\frac{a^2 \text{ cofin.}^2 T}{\text{sec.}^2 l} = +0,0322168.$$

$$\frac{a^2}{\text{sec.}^2 l} = +0,9970105;$$

$$\frac{R^2}{\sec.^2 l} = +361,5945100.$$

therefore

$$\text{Log. } v = 1,2744064.$$

$$\text{Log. } \frac{a' \cosin. T'}{\sec.^2 l} = 8,8971015;$$

$$\text{Log. } \frac{a'^2 \cosin.^2 T'}{\sec.^4 l} = 7,7942030;$$

$$\text{Log. } \frac{a'^2}{\sec.^2 l} = 9,9960553;$$

$$\text{Log. } \frac{R^2}{\sec.^2 l} = 2,5582207.$$

$$\frac{a' \cosin. T'}{\sec.^2 l} = -0,0789045;$$

$$\frac{a'^2 \cosin.^2 T'}{\sec.^4 l} = +0,0062259;$$

$$\frac{a'^2}{\sec.^2 l} = +0,9909580;$$

$$\frac{R^2}{\sec.^2 l} = +361,5935100.$$

therefore

$$\text{Log. } v' = 1,2767107.$$

hence

$$\text{Log. } \frac{v}{R} \text{ tang. } l = \text{Log. fin. } \lambda = 7,6030614,$$

$$\text{Log. } \frac{v'}{R} \text{ tang. } l' = \text{Log. fin. } \lambda' = 7,6452694;$$

$$\text{Log. cofin. } \lambda = 9,9999965,$$

$$\text{Log. cofin. } \lambda' = 9,9999958.$$

Now, as we know that R is very near the truth, and as v and v' encrease nearly in proportion to it, it is evident that a small alteration in the logarithms of the sines of λ and λ' , will not sensibly alter the sums of these logarithms, nor the logarithms of the cofines. We have therefore, as constant quantities,

$$\text{Log. } \frac{1}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'} = 0,0000077,$$

$$\text{Log. } \frac{\text{fin. } \lambda \cdot \text{fin. } \lambda'}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'} = 5,2483308,$$

$$\frac{1}{\operatorname{cosec} \lambda \cdot \operatorname{cosec} \lambda'} = 1,0000178,$$

$$\frac{\sin \lambda \cdot \sin \lambda'}{\operatorname{cosec} \lambda \cdot \operatorname{cosec} \lambda'} = 0,0000177.$$

Taking now the corrected values of T and T' , we have

$$\operatorname{Log} \sin E = \operatorname{Log} \frac{a \cdot \sin T}{R \cdot \operatorname{cosec} \lambda} = 9,9922193 - \operatorname{Log} R$$

$$\operatorname{Log} \sin E' = \operatorname{Log} \frac{a' \cdot \sin T'}{R \cdot \operatorname{cosec} \lambda} = 9,9966624 - \operatorname{Log} R$$

and, as before,

$$\operatorname{Log} S = 5,7529065 - \operatorname{Log} R \sqrt{R};$$

M being a little greater than it was before, R will be a little less than we found it.

Let

$$R = 19,015,$$

we have

$$\operatorname{Log} R = 1,2790963,$$

$$\operatorname{Log} R \sqrt{R} = 1,9186444,$$

Log. fin. E=8,7131230,

Log. fin. E=8,7175661,

Log. S=3,8342621,

S=6827",51,

$\frac{1}{2}S=3413",75=0^{\circ}56'53",75,$

Log. fin. $\frac{1}{2}S=8,2187804,$

Log. $\frac{2 \cdot \text{fin.}^2 \cdot \frac{1}{2}S}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'}=6,7385985,$

$\frac{2 \cdot \text{fin.}^2 \cdot \frac{1}{2}S}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'}=0,0005477,7,$

Cofin. $\Sigma=\frac{1-\text{fin. } \lambda \cdot \text{fin. } \lambda'-2 \cdot \text{fin.}^2 \cdot \frac{1}{2}S}{\text{cofin. } \lambda \cdot \text{cofin. } \lambda'}=0,9994523,2.$

therefore

	0	'	"
+M=	4	3	22,10
-E'=	2	59	29,20
+E=	2	57	39,43
-Σ=	1	53	46,75
ω=	0	0	0,22

R is still too great by a very small quantity. Let

$$R=19,0144,$$

$$\text{Log. } R=1,2790826,$$

$$\text{Log. } R\sqrt{R}=1,9186239,$$

we have

$$\text{Log. fin. } E=8,7131367,$$

$$\text{Log. fin. } E'=8,7175798;$$

$$\text{Log. } S=3,8342826,$$

$$S=6827'',83,$$

$$\frac{1}{2}S=3413,92=0^{\circ}56'53'',92,$$

$$\text{Log. fin. } \frac{1}{2}S=8,2188017.$$

$$\text{Log. } \frac{2. \sin.^2 \frac{1}{2}S}{\cosin. \lambda. \cosin. \lambda'}=6,7386411,$$

$$\frac{2. \sin.^2 \frac{1}{2}S}{\cosin. \lambda. \cosin. \lambda'}=0,0005478,24,$$

$$\text{Cofin. } \lambda=0,9994522,6.$$

therefore

$$\begin{array}{r}
 \text{+M} = -4 \quad 3 \quad 22,10 \\
 \text{--E'} = +2 \quad 59 \quad 29,54 \\
 \text{+E} = +2 \quad 57 \quad 39,76 \\
 \text{--}\Sigma = +1 \quad 53 \quad 47,11 \\
 \hline
 \omega = +0 \quad 0 \quad 0,09
 \end{array}$$

R therefore lies between the narrow limits of 19,015 and 19,0144. By means of a proportion, I find

$$R = 19,0144342.$$

I have therefore as follows :

$$\text{Log. R} = ,2790834 ;$$

$$\text{Log. R}\sqrt{R} = 9186251 ;$$

$$\text{Log. fin. E} = 8,7131359 ;$$

$$\text{Log. fin. E'} = 8,77790 ;$$

$$\text{Log. S} = 3,8342814 ;$$

$$S = 6827",81 ;$$

$$\frac{1}{2}S = 3413",9 = 0 \quad 56 \quad 53,9 ;$$

$$\text{Log. fin. } \frac{1}{2}S = 8,2188003.$$

H

$$\text{Log.} \frac{2 \sin^2 \frac{1}{2} S}{\cosin. \lambda \cosin. \lambda} = 6,7386383;$$

$$\frac{2 \sin^2 \frac{1}{2} S}{\cosin. \lambda \cosin. \lambda} = 0,0005478,2.$$

$$\text{Cofin. } \Sigma = 0,9994522,65.$$

	0	'	"
+M=	4	3	22,1
-E'=	2	59	29,5
+E=	2	57	39,7
-Σ=	1	53	47,1
ω'	=	0	0 0,0

From the above example it appears, that a difference, in the errors of two observations at above five months distance, of 20'', produces an error in the radius vector of little more than $\frac{3}{100}$ of the Earth's mean distance from the Sun. Now as the difference of the errors of two accurate observations does not exceed 15'' or 16'', from the known accuracy of Mr SLOP, we may conclude in the supposition of our Planet's orbit being circular, that its dif-

stance from the Sun is nearly 19,0144342, and that consequently its periodical revolution is 82 years and about 11 months.

As the angle z during an interval of five months is exceeded by S by only $\frac{7}{10}$ of a second, a quantity which has very little influence in the value of the radius vector, it will be unnecessary in other examples of this Planet to calculate z . Which circumstance, together with taking, by a simple proportion, the logarithms of the cosines of the heliocentric latitudes, reduces the calculation to very great simplicity.

Let us now take the observations of the 6th of October and 13th March. We have as follows :

		d.	h.	'	"
1781.	October	6.	17	7	51;
1782.	March	13.	6	28	58.
	Difference	157.	13	21	7.

$$t'' = 13612867''.$$

$$\text{Log. } t'' = 7,1339494;$$

$$\odot = \overset{s}{6} \overset{o}{14} \overset{'}{14} \overset{''}{58,9};$$

$$\odot' = 11 \ 23 \ 18 \ 47,6.$$

$$\text{Log. } a = 9,9992284;$$

$$\text{Log. } a' = 9,9979059.$$

Sum of the aberration and nutation for
Oct. 6. = -8,4; and for March 13. = -8,0.

Therefore

$$P = \overset{s}{3} \overset{o}{2} \overset{'}{54} \overset{''}{34,7};$$

$$P' = 2 \ 28 \ 50 \ 43,1;$$

$$M = -0 \ 4 \ 3 \ 51,6;$$

$$T = 3 \ 11 \ 20 \ 24,2.$$

$$T' = 8 \ 24 \ 28 \ 4,5.$$

$$\text{Log. fin. } T = 9,9914376.$$

$$\text{Log. fin. } T' = 9,9979723.$$

$$\text{Log. cofin. } \lambda = 9,9999965.$$

$$\text{Log. cofin. } \lambda' = 9,9999958.$$

$$\text{Log. fin. } E = 9,9906695 - \log. R.$$

$$\text{Log. fin. } E' = 9,9958824 - \log. R.$$

$$\text{Log. } S = 5,7474428 - \log. R\sqrt{R}.$$

Let us suppose

$$R = 19,0144342,$$

we have

$$\text{Log. } R = 1,2790834;$$

$$\text{Log. } R\sqrt{R} = 1,9186251.$$

$$\text{Log. fin. } E = 8,7115861.$$

$$\text{Log. fin. } E' = 8,7167990.$$

$$\text{Log. } S = 3,8288177.$$

therefore

$$\begin{array}{r}
 \textcircled{0} \quad ' \quad '' \\
 +M = -4 \quad 3 \quad 51,6 \\
 -E' = +2 \quad 59 \quad 10,3 \\
 +E = +2 \quad 57 \quad 1,9 \\
 -\Sigma = -1 \quad 52 \quad 21,8 \\
 \hline
 \omega = -0 \quad 0 \quad 1,2
 \end{array}$$

Let

$$R = 19,008,$$

we have

$$\text{Log. } R = 1,2789364,$$

$$\text{Log. } R\sqrt{R} = 1,91840464,$$

$$\text{Log. fin. } E = 8,7117331,$$

$$\text{Log. fin. } E' = 8,7169460.$$

$$\text{Log. } S = 3,8290382.$$

therefore

$$\begin{array}{r}
 \textcircled{0} \quad ' \quad '' \\
 +M = -4 \quad 3 \quad 51,6 \\
 -E' = +2 \quad 59 \quad 13,9 \\
 +E = +2 \quad 57 \quad 5,5 \\
 -\Sigma = -1 \quad 52 \quad 25,2 \\
 \hline
 \omega' = +0 \quad 0 \quad 2,6
 \end{array}$$

The proportion gives

$$R = 19,0124.$$

$$\text{Log. } R = 1,2790369.$$

$$\text{Log. } R\sqrt{R} = 1,9185554.$$

$$\text{Log. fin. } E = 8,7116326.$$

$$\text{Log. fin. } E' = 8,7168455.$$

$$\text{Log. } S = 3,8288874.$$

	°	'	"
+ M =	—4	3	51,6
— E' =	+2	59	11,4
+ E =	+2	57	3,0
— Σ =	—1	52	22,8
ω	= 0	0	0,0

The nine following combinations are such, that an error of 7" or 8" in each observation cannot produce, in the value of the radius vector, an error of above $\frac{3}{100}$ of the Earth's mean distance from the Sun.

The observations by Mr SLOP of
Oct. 5. 1781, and March 14. 1782, give
 $R=19,014434.$

Oct. 6. 1781, and March 13. 1782,
 $R=19,012400.$

Oct. 5. 1781, and March 3. 1782,
 $R=19,011196.$

Oct. 8. 1781, and March 11. 1782,
 $R=19,003736.$

Oct. 18. 1781, and March 10. 1782,
 $R=19,001396.$

Oct. 24. 1781, and March 4. 1782,
 $R=18,982314.$

Oct. 24. 1781, and March 3. 1782,
 $R=18,981850.$

Oct. 24. 1781. and March 14. 1782,
 $R=18,980485.$

Oct. 24. 1781, and March 13. 1782,

$$R=18,978509.$$

The medium of the above combinations gives

$$R=18,996258.$$

In some of the four following combinations, an error in one of the observations of less than $\frac{1}{10}$ of a second makes an alteration of $\frac{4}{100}$ in the radius vector. The results shew how much Mr Slop's observations may be depended on.

The observations of

Oct. 5. and 6. 1781, give

$$R=18,920860.$$

Dec. 22. and 23. 1781,

$$R=19,052614.$$

Aug. 22. and Sept. 29. 1781,

$$R=19,017040.$$

Jan 8. and 15. 1782,

$$R=18,941870.$$

Taking a medium we have

$$R=18,983096.$$

This, added to the medium of the former combinations, and the sum divided by 2, gives

R, or the distance of the new planet from the Sun, $=18,989677$;

that is, nearly *two thousand millions of miles.*

and

$R\sqrt{R}$, or the time of its periodical revolution, $=82,7516$.

that is, *eighty-two years* and something more than *nine months.*

H's motion in its orbit is therefore as follows :

			°	'	"
Annual,	-	-	4	21	1,3
Diurnal,	-	-	0	0	42,88
Horary,	-	-	0	0	1,79

The observations of 24th October 1781,
and 13th March 1782, give for

The Longitude of its Node,	^s	^o	'	"
	2	13	28	55,4
The Inclination of its Orbit	}	o	o	48
to the Ecliptic,				7,4

Those of 24th October 1781, and 3d March
1782.

Long. Nod.	-	^s	^o		
		2	13	29	59,3
Incl. Orb.	.	-	o	o	48 11,7

The medium of these two gives

Long. Nod.	.	^s	^o	'	"
		2	13	29	27,3
Incl. Orb.	-	-	o	o	48 9,5

The above theory agrees extremely well
with the longitudes observed by Mr SLOP
with the Mural Quadrant. The errors of
most of them do not come the length of
5", and none exceed 7" excepting those of
October 18th, January 17th, and Febru-
ary 1st. Though this does not warrant

us to say that the New Planet's orbit is *circular*, it is beyond dispute that its distance from the sun differed very little during the space of above five months, and that that distance was nearly 18,989677.

Two other Series of good observations, at the distance of two or three years from that which we have made use of, and from each other, will give us by a similar procedure two other radii vectores.

By means of the three radii vectores thus found, we shall be able in a few years to determine with tolerable accuracy all the elements of the *elliptical* orbit of our Planet.

Though the inclination of its orbit above determined agrees pretty well with the latitudes observed during the five months referred to, it cannot be expected to be very exact. From a comparison of the latitudes of August and September

1781 with those of August and September 1782, the Planet seems to have advanced in latitude something more than 3'. Whence it would appear that the inclination of its orbit surpasses one degree.

The longitudes of August and September 1781, compared with those of August and September 1782, shew that the Planet's velocity is increasing. It would seem, therefore, to be advancing towards the perihelion.

SECTION VII.

Mr SLOP's Theory, &c.

FROM two interpolated places, the one 12 h. before, and the other 12 h. after, the Planet's opposition to the Sun on December 21st 1781, Mr SLOP found

The Radius Vector = 18,9894

and, from a comparifon of the obfervations of 15th May, 17th July, and 18th October 1781 with thofe of 13th and 14th March 1782,

Long. Nod. $= 2^s 14^{\circ} 24' 20''$,

Incl. Orb. $= 0^{\circ} 0' 51'' 41$.

This theory gives for the errors of the obfervations made at Pifa from 22d Auguft 1781 to 14th March 1782 inclusive, as follows :

1781.	<i>Err. in Long.</i>	<i>Err. in Lat.</i>
Aug. 22.	+0 34,7	—0 7,7
23.	+0 28,4	—0 11,7
24.	+0 29,3	—0 17,8
26.	+0 31,0	—0 10,0
27.	+0 31,4	—0 6,6
28.	+0 26,3	—0 9,4
29.	+0 28,3	—0 10,1
30.	+0 30,9	—0 9,3
Sept. 1.	+0 24,2	—0 5,9
2.	+0 20,2	—0 6,1
5.	+0 20,7	—0 7,0
11.	+0 20,4	—0 6,0
15.	+0 15,6	—0 6,1
18.	+0 9,0	—0 12,4
24.	+0 11,7	—0 8,6
29.	+0 8,3	—0 8,0

1781.		<i>Err. in Long.</i>	<i>Err. in Lat.</i>
Nov.	11.	+ 0 3,4	+ 0 6,5
Oct.	5.	+ 0 6,9	— 0 11,9
	6.	+ 0 7,1	— 0 4,5
	8.	+ 0 5,9	— 0 3,7
	9.	+ 0 5,3	— 0 8,4
	18.	+ 0 11,8	— 0 3,3
	24.	+ 0 1,3	— 0 2,9
Dec.	4.	— 0 4,6	— 0 2,6
	6.	— 0 4,9	— 0 2,8
	8.	— 0 4,4	— 0 7,8
	13.	— 0 2,7	— 0 7,4
	14.	— 0 0,1	— 0 8,6
	22.	+ 0 2,0	— 0 2,3
	23.	+ 0 2,0	— 0 0,9
	27.	— 0 1,7	— 0 0,9
	28.	— 0 6,7	— 0 2,3
1782.			
Jan.	1.	— 0 2,0	— 0 3,8
	2.	— 0 7,5	— 0 4,2
	4.	— 0 1,1	— 0 2,8
	6.	— 0 1,3	— 0 3,9
	8.	+ 0 0,7	— 0 3,5
	9.	+ 0 5,7	— 0 2,3
	11.	+ 0 2,3	— 0 3,2
	12.	+ 0 1,1	— 0 0,1
	13.	+ 0 3,4	— 0 2,1
	14.	+ 0 1,6	— 0 1,5
	15.	— 0 0,5	— 0 0,9

		<i>Err. in Long.</i>	<i>Err. in Lat.</i>
1782.			
Jan.	17.	—o 11,0	+o 2,0
	19.	—o 3,9	—o 0,6
	20.	+o 3,4	—o 3,1
	22.	+o 0,2	—o 2,6
	28.	—o 1,7	—o 2,7
Feb.	1.	+o 9,5	—o 1,4
	4.	+o 1,9	+o 0,3
	11.	+o 0,1	+o 1,4
	15.	+o 2,1	—o 1,3
	16.	+o 3,8	—o 3,3
	18.	—o 0,5	—o 2,8
	19.	+o 0,7	—o 2,8
	20.	+o 4,4	—o 0,2
	26.	+o 5,6	—o 0,8
	27.	—o 6,0	+o 3,5
	Mar. 3.	+o 1,2	+o 1,0
	4.	+o 1,2	+o 2,3
	5.	—o 0,4	+o 1,0
	6.	—o 4,9	—o 2,5
	8.	+o 5,2	+o 4,1
	10.	—o 5,7	+o 3,7
	11.	—o 2,5	+o 1,0
	13.	—o 5,7	—o 1,4
	14.	—o 6,9	o 0,0

THE END.



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